22087208

International Baccalaureate® Baccalauréat International Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 2

Thursday 8 May 2008 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Determine the first three terms in the expansion of $(1-2x)^5(1+x)^7$ in ascending powers of x.

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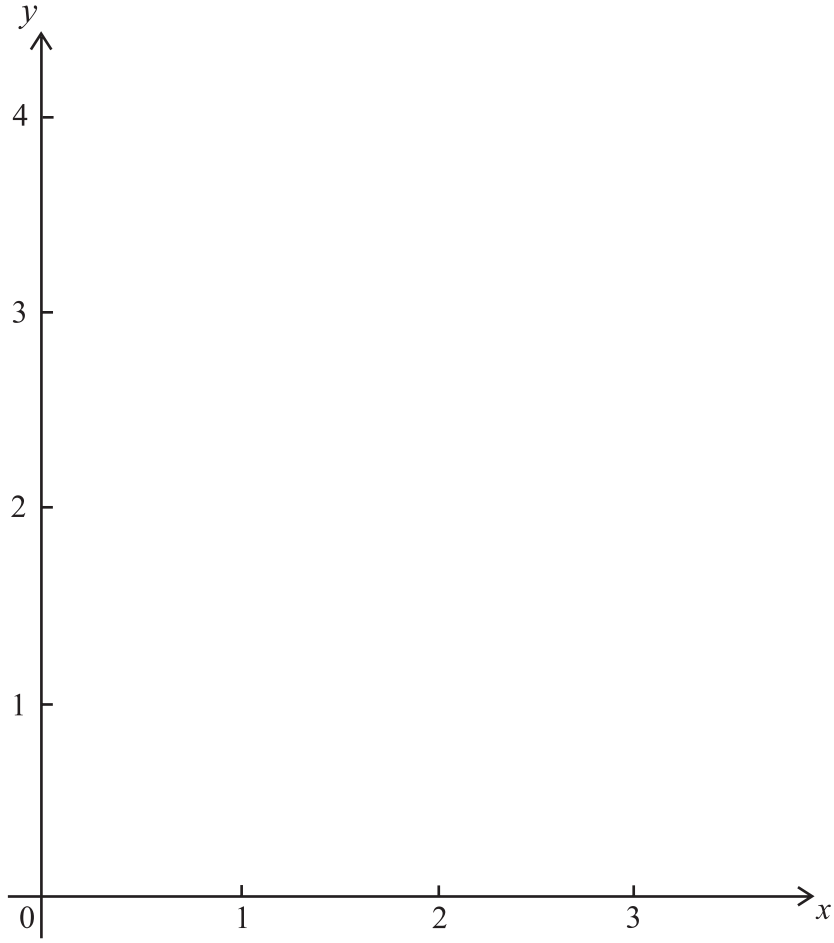
2. *[Maximum mark: 6]*

(a) Sketch the curve $f(x) = |1 + 3\sin(2x)|$, for $0 \le x \le \pi$. Write down on the graph the values of the x and y intercepts. *[4 marks]*



(b) By adding **one** suitable line to your sketch, find the number of solutions to the equation $\pi f(x) = 4(\pi - x)$. *[2 marks]*

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3. *[Maximum mark: 6]*

A ray of light coming from the point $(-1, 3, 2)$ is travelling in the direction of vector $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ and meets the plane $\pi : x + 3y + 2z - 24 = 0$.

Find the angle that the ray of light makes with the plane.

4. *[Maximum mark: 6]*

A company produces computer microchips, which have a life expectancy that follows a normal distribution with a mean of 90 months and a standard deviation of 3.7 months.

(a) If a microchip is guaranteed for 84 months find the probability that it will fail before the guarantee ends. *[2 marks]*

(b) The probability that a microchip does not fail before the end of the guarantee is required to be 99 %. For how many months should it be guaranteed? *[2 marks]*

(c) A rival company produces microchips where the probability that they will fail after 84 months is 0.88. Given that the life expectancy also follows a normal distribution with standard deviation 3.7 months, find the mean. *[2 marks]*

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5. *[Maximum mark: 6]*

Find the vector equation of the line of intersection of the three planes represented by the following system of equations.

$$\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ -14x - 23y + 13z &= 5 \end{aligned}$$

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6. *[Maximum mark: 6]*

Find the gradient of the tangent to the curve $x^3y^2 = \cos(\pi y)$ at the point $(-1, 1)$.

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7. *[Maximum mark: 6]*

A system of equations is given by

$$\cos x + \cos y = 1.2$$
$$\sin x + \sin y = 1.4\,.$$

(a) For each equation express y in terms of x. *[2 marks]*

(b) **Hence** solve the system for $0 < x < \pi$, $0 < y < \pi$. *[4 marks]*

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8. *[Maximum mark: 6]*

Only two international airlines fly daily into an airport. UN Air has 70 flights a day and IS Air has 65 flights a day. Passengers flying with UN Air have an 18 % probability of losing their luggage and passengers flying with IS Air have a 23 % probability of losing their luggage. You overhear someone in the airport complain about her luggage being lost.

Find the probability that she travelled with IS Air.

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9. *[Maximum mark: 6]*

By using an appropriate substitution find

$$\int \frac{\tan(\ln y)}{y}\,\mathrm{d}y,\ y > 0\,.$$

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10. *[Maximum mark: 7]*

Find, in its simplest form, the argument of $(\sin\theta + \mathrm{i}(1-\cos\theta))^2$ where θ is an acute angle.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 13]*

The lifts in the office buildings of a small city have occasional breakdowns. The breakdowns at any given time are independent of one another and can be modelled using a Poisson Distribution with mean 0.2 per day.

(a) Determine the probability that there will be exactly four breakdowns during the month of June (June has 30 days). *[3 marks]*

(b) Determine the probability that there are more than 3 breakdowns during the month of June. *[2 marks]*

(c) Determine the probability that there are no breakdowns during the first five days of June. *[2 marks]*

(d) Find the probability that the first breakdown in June occurs on June 3^{rd}. *[3 marks]*

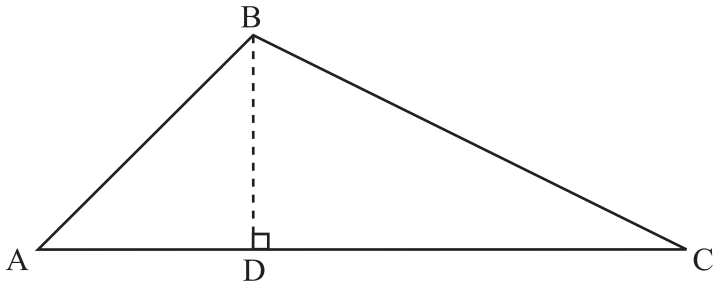
(e) It costs 1850 Euros to service the lifts when they have breakdowns. Find the expected cost of servicing lifts for the month of June. *[1 mark]*

(f) Determine the probability that there will be no breakdowns in exactly 4 out of the first 5 days in June. *[2 marks]*

12. *[Total mark: 20]*

Part A *[Maximum mark: 12]*

In triangle ABC, $BC = a$, $AC = b$, $AB = c$ and [BD] is perpendicular to [AC].

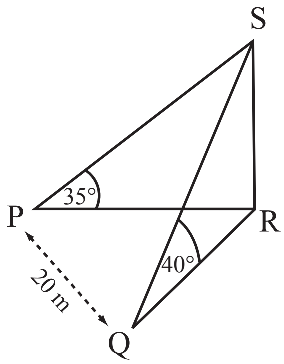


(a) Show that $CD = b - c\cos A$. *[1 mark]*

(b) **Hence**, by using Pythagoras' Theorem in the triangle BCD, prove the cosine rule for the triangle ABC. *[4 marks]*

(c) If $A\hat{B}C = 60^\circ$, use the cosine rule to show that $c = \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2}$. *[7 marks]*

Part B *[Maximum mark: 8]*



The above three dimensional diagram shows the points P and Q which are respectively west and south-west of the base R of a vertical flagpole RS on horizontal ground. The angles of elevation of the top S of the flagpole from P and Q are respectively 35° and 40°, and $PQ = 20$ m.

Determine the height of the flagpole.

13. *[Maximum mark: 13]*

A family of cubic functions is defined as $f_k(x) = k^2x^3 - kx^2 + x$, $k \in \mathbb{Z}^+$.

(a) Express in terms of k

(i) $f_k'(x)$ and $f_k''(x)$;

(ii) the coordinates of the points of inflexion P_k on the graphs of f_k. *[6 marks]*

(b) Show that all P_k lie on a straight line and state its equation. *[2 marks]*

(c) Show that for all values of *k*, the tangents to the graphs of f_k at P_k are parallel, and find the equation of the tangent lines. *[5 marks]*

14. *[Maximum mark: 14]*

$z_1 = (1 + \mathrm{i}\sqrt{3})^m$ and $z_2 = (1 - \mathrm{i})^n$.

(a) Find the modulus and argument of z_1 and z_2 in terms of *m* and *n*, respectively. *[6 marks]*

(b) **Hence**, find the smallest positive integers *m* and *n* such that $z_1 = z_2$. *[8 marks]*